HELSINKI UNIVERSITY OF TECHNOLOGY **ABSTRACT OF THE LICENTIATE'S THESIS**

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Solution of dense systems of linear equations in electromagnetic scattering calculations

Tiheiden lineaaristen yhtälöryhmien ratkaiseminen sähkömagneettisissa sirontalaskuissa

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The scattering of electromagnetic radiation from particles is governed by an integral equation over the volume of the particle. The discretization of integral equations leads to dense systems of linear equations. In this thesis the efficient solution of these systems is studied.

The integral equations of scattering are reviewed. The discretized integral equation is related to another computational technique, the discrete-dipole approximation (DDA), where the particle is modeled by interacting electric dipoles.

In this work, several iterative methods were implemented and tested on the dense system of linear equations arising from the DDA method. The iterative method QMR and its block version were found to be the most efficient methods. The iterative methods converged in relatively few iterations. When the discretization of the particle was refined, the number of iterations remained constant. The eigenvalues of some coefficient matrices were calculated. In some cases the convergence of iterative methods could be accurately estimated from the eigenvalue spectrum.

The computationally most expensive part in the iterative solvers is the computation of the matrix-vector product. In addition to the direct calculation of the matrix entries, two fast algorithms for the matrix-vector product, the fast Fourier transform and the fast multipole method, were implemented. The potential expansions and the translation formulas for these expansions were derived for the fast multipole algorithm.

The fast Fourier transform is very efficient it the dipoles are located densely on a regular lattice. We were able to solve equations with more that 400 000 unknowns with the FFT method. If the geometry is sparse and irregular, the fast multipole method becomes the fastest method to compute the matrix-vector product.